Nonstationary Response of Nonlinear Systems with a Compact Analytical Form of Data-Based Excitation Processes

Andrew W. Smyth
Assistant Professor
School of Engineering & Applied Science
Columbia University
New York, New York 10027-6699

Sami F. Masri
Professor
School of Engineering
University of Southern California
Los Angeles, California 90089-2531

INTRODUCTION:

The topic of stochastic response of dynamic systems has been an area of considerable interest for some time in the analysis of risk and structural reliability. A substantial reference list of work in this area can be found in Roberts and Spanos (1998). The authors, in previous work (Masri *et al.* (1998) and Smyth (1998)), have developed a method which can analyze the response of linear multi-degree-of-freedom systems to completely general data-based nonstationary excitations in a highly efficient and analytical form. This was demonstrated with successful application to a model structure subjected to an ensemble of ground-motion recordings from the 1994 Northridge (California) Earthquake. The authors have now extended this work to nonlinear system response by using equivalent linearization techniques (Smyth and Masri, 2001).

OVERVIEW OF NONSTATIONARY EXCITATION COMPACT REPRESENTATION:

The fundamental starting point for the development of the new technique is in the compact analytical data approximation used for the nonstationary excitation. After applying Karhunen-Loeve spectral decomposition to the measured excitation process covariance matrix $\mathcal{C}_k(t_1, t_2)$, and least-squares fitting the eigenvectors with orthogonal polynomials, the excitation data is condensed into the following approximate form:

$$\hat{\mathcal{C}}_k(t_1, t_2) = \sum_{i=1}^{k} \lambda_i \sum_{j=0}^{m_i - 1} \sum_{\ell=0}^{m_i - 1} H_{ij} H_{i\ell} T_j(t_1') T_\ell(t_2'), \tag{1}$$

where the λ_i's are the truncated series of eigenvalues, the $T_j(t_1')$'s are Chebyshev polynomials, and the H's are weighting coefficients for the Chebyshev polynomials in the least-squares fitting of the eigenvectors. For additional details the reader is referred to Masri *et al.* (1998).

This compact analytical expression is in such a form which allows the closed-form solution of the nonstationary response problem for general (i.e., without assuming proportional damping) linear systems of the form: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$.

STATISTICAL LINEARIZATION AND NONSTATIONARY EXCITATION:

In the case of nonlinear system response, because it is difficult to obtain probabilistic response solutions for nonstationary (and even stationary) excitations, researchers have turned to approximating the nonlinear system as an *equivalent linear* system. The reader may refer to *Random Vibration and Statistical Linearization* (Roberts and Spanos, 1990) for a comprehensive treatment of the subject.

Consider the following nonlinear *n* degree of freedom equation of motion

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) + \mathbf{g}(\mathbf{q}(t), \dot{\mathbf{q}}(t)) = \mathbf{F}(t) \tag{2}$$

where $\mathbf{g}$ is a nonlinearity dependent upon the system displacements and velocities, and $\mathbf{F}(t)$ is a random vector process of length n. After some manipulation, this nonlinear system can be written in its equivalent linear state-space form (see Roberts and Spanos, 1990):

$$\dot{\mathbf{z}} = \mathbf{G}(t)\mathbf{z} + \mathbf{f} \tag{3}$$

where

$$\mathbf{G}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}(\mathbf{K} + \mathbf{K_e}) & -\mathbf{M}^{-1}(\mathbf{C} + \mathbf{C_e}) \end{bmatrix} \quad \text{and} \quad \mathbf{f} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{F_0} \end{bmatrix} \tag{4}$$

where $\mathbf{C_e}$ and $\mathbf{K_e}$ are *time-dependent* equivalent linear terms which account for the nonlinear $\mathbf{g}$; this contrasts with stationary statistical linearization problems where these matrices are constant.

The goal is to get a second order probabilistic description of the response state $\mathbf{z}$, i.e. $\mathbf{V}(t) \equiv E\{\mathbf{z}(t)\mathbf{z}^T(t)\}$. The differential equation which defines the evolution of $\mathbf{V}(t)$ is

$$\dot{\mathbf{V}} = \mathbf{G}(t)\mathbf{V} + \mathbf{V}\mathbf{G}^T(t) + \mathbf{U}(t) + \mathbf{U}^T(t) \qquad \text{where} \qquad \mathbf{U}(t) = E\{\mathbf{z}\mathbf{f}^T\} \tag{5}$$

After incorporating the compact form of the excitation process one may write $\mathbf{U}(t)$ as

$$\mathbf{U}(t) = \sum_{i=1}^{k} \lambda_i \sum_{j=0}^{m_i-1} \sum_{\ell=0}^{m_i-1} H_{ij} H_{i\ell} T_j(t') \mathbf{U}^{(\ell)}(t) \tag{6}$$

and after some further manipulation $\mathbf{U}^{(\ell)}(t)$ can be expressed as

$$\dot{\mathbf{U}}^{(\ell)}(t) = \mathbf{G}(t)\mathbf{U}^{(\ell)}(t) + \mathbf{R}T_\ell(t') \tag{7}$$

where $\mathbf{R}$ is a very simple constant matrix which defines to which degrees of freedom the force is applied, e.g., in the single input case

$$\mathbf{W_f}(t_1, t_2) \equiv E\{\mathbf{f}(t_1)\mathbf{f}^T(t_2)\} = \mathbf{R}\mathcal{C}_{ss}(t_1, t_2) \tag{8}$$

This new solution is highly efficient, contrasting with an analogous solution by Roberts and Spanos (1990) which is limited to a restrictive class of nonstationary excitations, and involves 3-index terms $\mathbf{U}^{(ijk)}(t)$ which increases the number of terms in the nested summations in Eq. (6).

REFERENCES:

1. Masri, S.F., Smyth, A.W., and Traina, M.-I., (1998), "Probabilistic Representation and Transmission of Nonstationary Processes in Multi-Degree-of-Freedom Systems," *Jo. of Applied Mechanics, ASME,* Vol. 65, June, pp. 398–409.
2. Roberts, J.B., and Spanos, P.D., (1990), *Random Vibration and Statistical Linearization,* John Wiley & Sons, New York, NY.
3. Sakata, M., and Kimura, K., (1980), "Calculation of the Non-Stationary Mean Square Response of a Non-Linear System Subjected to Non-White Excitation," *Jo. of Sound and Vibration,* vol. 73, no. 3, pp. 333-343.
4. Smyth, A.W., (1998), *Experimental and Analytical Studies in Nonlinear System Identification and Modeling for Structural Control,* Ph.D. Thesis, University of Southern California, Los Angeles, CA.
5. Smyth, A.W., and Masri, S.F., (2001), "Nonstationary Response of Nonlinear Systems Using Equivalent Linearization with a Compact Analytical Form of the Excitation Process," *Probabilistic Engineering Mechanics,* [accepted for publication].