

LOTKA-VOLTERRA SYSTEM IN A RANDOM ENVIRONMENT

M.F.DIMENTBERG

(Mechanical Engineering Department, Worcester Polytechnic Institute, Worcester, MA, USA)

Classical Lotka-Volterra (LV) model for oscillatory behavior of population sizes of two interacting species (predator-prey or parasite-host pairs) is conservative. This means that sustained oscillations are regarded as being completely controlled by initial conditions, which are “never forgotten” by the system. Such nonrobust models have certain well-known drawbacks. In particular they imply unrealistically high sensitivity of the system’s behavior to environmental variations. The models with internal dissipation and sustained external excitation may be more adequate in this respect. Thus, a generalized stochastic LV model is considered, with the equation for preys’ reproduction containing the following additional terms: i) quadratic “damping” term which describe self-limitation of their population due to interspecies competition; and ii) term with white-noise random variations of the preys’ reproduction factor which simulates variations in the environmental conditions. This model had been introduced in [1] and studied by Svirejev and Logofet [2], who presented stochastic stability analysis for the linearized equations. The potential instability in the mean square had been interpreted as implying the potential extinction of the populations.

An exact solution is obtained in the present work for the corresponding Fokker-Planck-Kolmogorov (FPK) equation for stationary joint probability densities function (p.d.f.) of the population sizes. This solution indicates that both population sizes are independent gamma-distributed stationary random processes. Increasing level of the environmental variations does not lead by itself to extinction of the populations. However, it may lead to major change(s) in the p.d.f.(s) of the population(s): the most probable population size(s) may drop to zero, where the corresponding p.d.f. has an integrable singularity. Such a pattern in the p.d.f.(s) corresponds to an on-off intermittent behavior, whereby population size of predators, or both population sizes may experience very rare and violent short pulses (outbreaks) while remaining on a very low level most of the time. (This expectation based on the analysis of the solution for the p.d.f.'s is confirmed by results of direct Monte-Carlo simulation). This intermittency is somewhat different from the one observed in known cases with dynamic instability of the linearized model, where sporadic oscillations may be observed. In the present case the observed outbreaks contain basically single isolated high peaks, so that this kind of intermittency may be called “quasistatic”. Parametric study of the solution indicates that the LV system is especially prone to the intermittent behaviour in the proximity of the bifurcation point, which corresponds to the extinction of predators (or parasites). It may be expected that such a quasistatic intermittency may be typical in general for systems with randomly varying parameters in the vicinity of the saddle-node bifurcation points.

The on-off intermittency is described analytically by using the obtained solutions for the p.d.f.'s, which acquire integrable singularity(ies) at zero for high excitation intensity. Thus, relative stay time of the gamma-distributed process above its mean level is calculated and is shown to be really small indeed in the case of intermittency. Theory of excursions of random functions is also applied indicating that the excursions may be very rare indeed and should have typical shape of short pulses. Furthermore, p.d.f. of the peaks in the predators’ population size is calculated. Special emphasize is made on the case, where the “mean” system is close to its bifurcation point corresponding to the threshold for extinction of predators due to the high interspecies competition between preys. (This is the bifurcation point of the saddle-node type). In this case high and rare outbreaks in the number of predators (or parasites) are to be expected. Such outbreaks are really observed in Monte-Carlo simulation results for the stochastic LV

system. The latter may be regarded therefore as an adequate model for the phenomena actually observed in forests in Quebec (Canada), where seven outbreaks of budworms were reported since 1710. Other examples of population data are presented as well, which can be described by the above theory. Using theory of first-passage probability analysis, the results of this study can be used to predict the outbreaks in populations as well as for probabilistic forecasts for potential extinction of rare species.

REFERENCES

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