Simulation of Dynamical Systems via Adaptive Importance Sampling using Discrepancy Measures

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While there are many procedures for analyzing the response of stochastic systems, most are restricted to systems of small dimension where averaging or perturbation methods apply. With white noise excitation processes, analytical solution of systems with many degrees of freedom is still limited to linear systems, nonlinear systems with assumed normally distributed response (stochastic equivalent linearization), and a special class of conservative nonlinear systems. More general stochastic systems have been examined through numerical solution of their respective Fokker-Planck equations by several methods, including finite elements, finite differences, path integral methods, and cell mapping. These methods have allowed insight into some problems of engineering interest, though, at the present time, they are limited to no more than dimension four.

In engineering practice, however, the most pressing problems are complex, nonlinear and modeled by systems having many degrees of freedom. These problems are most often simulated using standard large-scale finite element (FE) codes, and the random component can only be tackled through Monte Carlo simulation. In cases where the response distribution to be estimated is multimodal or otherwise complex, or when low probability regions (such as failure probabilities) must be accurately characterized, direct Monte Carlo simulation (dMCS) methods can consume resources to the point of being intractable. The choice, then, is to either produce millions or billions of realizations — clearly a cost-prohibitive option for large-scale models typical in FE analyses — or to develop methods to limit the required number of realizations for a given accuracy level.

For static problems, various weighted sampling methods have been standard practice since the early days of Monte Carlo methods. Application of these methods to dynamical systems with continuous random excitation is rather difficult since the excitation random variable space is often of extremely high dimension. One can, however, gain important insights from importance and stratified sampling techniques. The common ground is that the sample distribution is artificially modified in order to reduce error, but with a corresponding change in the "weight" of a sample point to retain unbiased estimates. Whereas dMCS uses an equal weight for all samples, the ideal might be samples uniformly distributed throughout the "interesting" portion of the phase space with weights producing the density function. Standard variance reduction techniques can have such an effect for static problems, but dynamic systems are more problematic.

Johnson (1997) and Johnson *et al.* (1999) proposed the use of *discrepancy sensitivity* as a measure of the contribution of individual sample points, in static or dynamic problems, to distribution uniformity. In other terms, the discrepancy sensitivity gives a sense of the information content in a sample point, whether it contributes new information or is largely redundant. This sensitivity, then, may be used to decide whether to continue using that sample point (realization).

For example, the discrepancy sensitivities in one instantaneous distribution of samples are shown in Fig. 1. The sensitivities near the mean are positive, indicating that extraneous samples are located there, and sensitivities in the "tails" are large negative values, implying that those samples contain significant information. This is exactly the type of indicator required for characterizing the importance of realizations in a stochastic dynamics problem. Further, it is worth noting that this measure of importance is neither dependent on the dimensionality of the system nor any physical meaning of the states (such as an energy measure or similar property).

By using a genetic algorithm (GA) to pick and choose which realizations to keep and which to "mate" to develop new sample points, the realizations that contain less information are suppressed (but not completely eliminated) and those of greater importance are enhanced and encouraged. This method is related to the dual approaches of *splitting* and *Russian roulette* used in early in Monte Carlo development for neutron transport problems and recently explored by Pradlwarter and Schuëller (1997).

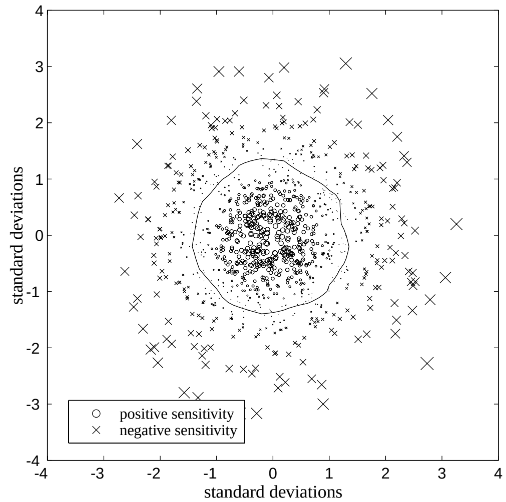

Figure 1: Discrepancy sensitivity for a bivariate Gaussian distribution. The solid line represents an approximate contour of zero sensitivity. The size of the markers indicate sensitivity magnitude.

An example of a GA-directed Monte Carlo simulation was reported by Johnson (1997) for a bimodal Duffing oscillator subjected to white noise excitation. The method was able to accurately characterize cumulative distribution functions to approximately the same probability level as a dMCS with two orders of magnitude more realizations. Application of GA-directed MCS to a more complex system, a 120-state model of a 12-story building, is in progress. Though the algorithm is not yet fully developed for larger systems, Fig. 2 shows an early result where a GA-directed MCS estimates a CDF of the roof displacement during a simulated earthquake down to probability levels about an order of magnitude lower than dMCS. This should be viewed as an indication of potential, as it is a work in progress, and not as a final result.

While discrepancy sensitivity and genetic algorithms show promise in reducing computational effort in characterizing uncertainty of stochastic dynamical systems, they have yet to be fully explored. As these methods are based on probabilistic principles, the potential exists for developing error bounds or confidence intervals, such as is available with direct Monte Carlo techniques, but that are generally unavailable for many uncertainty estimation techniques.

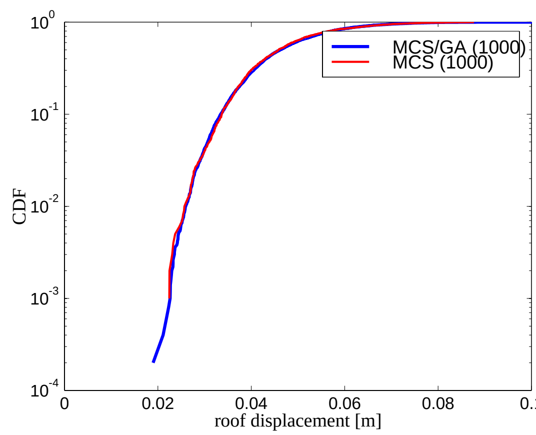

Figure 2: Cumulative distribution function of roof displacement of a 12-story building when subject to a nonstationary filtered white noise ground motion.

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