Spectral Description and Simulation of Non-stationary Random Processes by Hilbert Transform

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ABSTRACT

A new method is proposed for characterization and simulation of a nonstationary random process based on samples of the process. The theoretical background is that of the Huang Hilbert Transform (HHT) and Empirical Mode Decomposition EMD (Huang et al 1998). Samples of a random process X(t) can be decomposed into a summation of modal functions which after a Hilbert transform describe the amplitude and frequency changes with time. A Hilbert spectrum is then defined to describe the time-varying spectral content of the sample process. The expected value (or ensemble average) of the spectrum characterizes the spectral content of the process. There are many advantages using the Hilbert spectrum over those based on a Fourier transform method when dealing with nonstationary and nonlinear random processes (Huang et al, 1998).

For a vector (multivariate) nonstationary process, the HHT+EMD based spectral description is extended herein via a cross Hilbert spectrum between any two components which is in general a complex quantity. The real part describes the in-phase spectral content between the two components as function of time and the imaginary part the out-of-phase spectral content between the two components. For many real world problems such as earthquake excitations, only one sample function is available and ensemble average of the Hilbert spectrum is not possible. For such situations in practical application, the Hilbert spectra based on the sample are smoothed and used as the spectrum for the underlying random process. The advantages in using a Hilbert spectrum based spectral description over other current available methods, such as evolutionary power spectral density function, in engineering applications are indicated.

A method of simulation of nonstationary random processes based on observed sample functions is then proposed based on the Hilbert spectral characterization. Both univariate and multivariate processes can be simulated without difficulty. The simulated processes are Gaussian and will have the Hilbert spectra equal to the target spectra. Unlike current procedures such as those based on the evolutionary process, no assumptions of any functional forms for the spectra are necessary for parameter estimation, which are unknown a priori. The method then is applied to spectral characterization and simulation of multivariate earthquake ground motions. Results show that the Hilbert spectra give a clear description of the spectral content and correlation change with time. The simulated samples have the desired frequency and amplitude variation with time. They prove the validity and great potential for engineering applications of the proposed method when dealing with non-stationary, nonlinear random processes.

Huang et al " The empirical mode decomposition and the Hilbert spectrum for nonlinear, nonstationary time series analysis", Proc. R. Soc. Lond. A (1998) 454, 903-995.